Name: ______________________ Date: ___________________

Math 10F&PC Chapter 4 Roots and Powers

4.2 Irrational Numbers

> more than
≥ more than or equal
< less than
≤ less than or equal

There are a few categories that we sort numbers into:

Natural Numbers: 1, 2, 3, 4, ... any number ≥ 1

Whole Numbers: 0, 1, 2, 3, 4, ... any number ≥ 0

Integers: ...,-2, -1, 0, +1, +2, ...

We also have:

Rational Numbers: numbers that can be written as a fraction $\frac{m}{n}$ Decimal form terminates, or keeps repeating

0.7 $\boxed{R}$ $\frac{3}{7}$ $\boxed{R}$ $0.7\overline{11}$ $\boxed{R}$ 0.424 $\boxed{R}$

0.715156131217 $\boxed{I}$

Irrational Numbers: numbers that are not rational (can't be written as a fraction) $\frac{m}{n}$ $n \neq 0$
Decimal form does not terminate or repeat

$\pi \rightarrow \boxed{I}$ $\sqrt{2} \rightarrow \boxed{I}$ $\sqrt{\frac{1}{3}} = \boxed{I}$ $\sqrt{\frac{1}{4}} = \boxed{R}$

For radicals x: If radicand (x) is NOT a perfect number for index n, then the radical is IRRATIONAL

$\sqrt[n]{x}$

Ex. $\sqrt[4]{\frac{1}{16}} = \boxed{R}$ Ex:- $\sqrt{\frac{1}{9}} = \frac{1}{3} = \boxed{R}$ $\sqrt[3]{\frac{1}{9}} = \boxed{I}$

$\sqrt[4]{\frac{1}{8}} = \boxed{I}$

a. How are radicals that are rational numbers different from radicals that are not rational numbers? radicals that are rational numbers have terminating or repeating decimals.

b. Which of these radicals are rational numbers? Which are not rational numbers? Explain how you know?

a) $\sqrt{\frac{49}{16}} = \frac{\sqrt{49}}{\sqrt{16}} = \boxed{\frac{7}{4}}$ b) $\sqrt[3]{-30} = -3.1072\ldots$ c) 1.21

R ⇒ it is in the form of $\frac{m}{n}$ or $=\frac{7}{4}$ or 1.75

I ⇒ Not perfect cube
No repeated
No terminate

R ⇒ terminating decimal

Since irrational numbers have decimal representation that neither terminate nor repeat. It is impossible to express an irrational number *exactly* using decimals.
The only way to express an irrational number exactly is to use radicals.

$\sqrt{2}$ I Radical

$\sqrt{2} = 1.41421356\ldots$

Example: Tell whether each number is rational or irrational. Explain how you know.

a) $-\frac{3}{5} \Rightarrow \boxed{R}$

in form of $\frac{m}{n}$ fraction

b) $\sqrt{14} \Rightarrow$ I

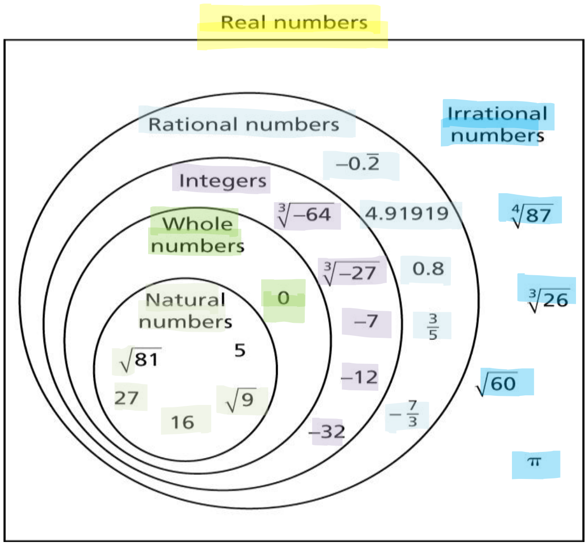
$\sqrt{14} = 3.741657\ldots$

N R
N T

c) $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \boxed{\frac{2}{3}}$

R in form $\frac{m}{n}$ Fraction

Together, the rational numbers and irrational numbers form the set of *real numbers*. This diagram shows how these number systems are related.



Number Systems
Natural (N) – 1, 2, 3, . . .
Whole (W) – 0, 1, 2, . . .
Integers (I) – −3, −2, −1, 0, 1, 2, 3,
Rational (Q) – Can be written as $\frac{a}{b}$, $b \neq 0$ (i.e. terminating or repeating decimals)
Irrational $\overline{Q}$ –Cannot be written as $\frac{a}{b}$, $b \neq 0$ (i.e. non–terminating, non–repeating decimals)Best example is π.
Real Numbers – the union of rational and irrational numbers. If the number exists, it is a Real Number.

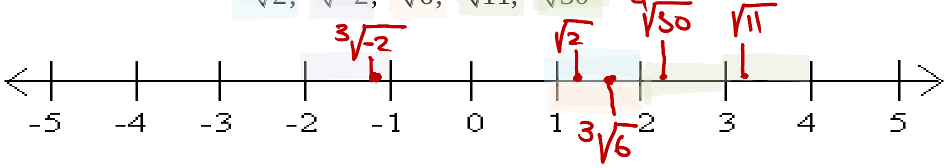
number
Real Non Real

Example 2: **Classify the following.**

1. $\sqrt{49}$ $\boxed{R}$ — N
2. $-\sqrt{2}$ $\boxed{I}$ $= -1.4142\ldots$
3. $-0.\overline{5}$ $\boxed{R}$
4. $\sqrt{\frac{1}{4}}$ $\boxed{R}$
5. 0.358 $\boxed{R}$
6. 0.18283848 . . . $\boxed{I}$
7. π $\boxed{I}$
8. $\frac{0}{2}$ $\boxed{R}$
9. $\frac{5}{0}$ No real number

Example 3: *Use a number line to order these numbers from least to greatest.*

$\sqrt{2}$, $\sqrt[3]{-2}$, $\sqrt[3]{6}$, $\sqrt{11}$, $\sqrt[4]{30}$



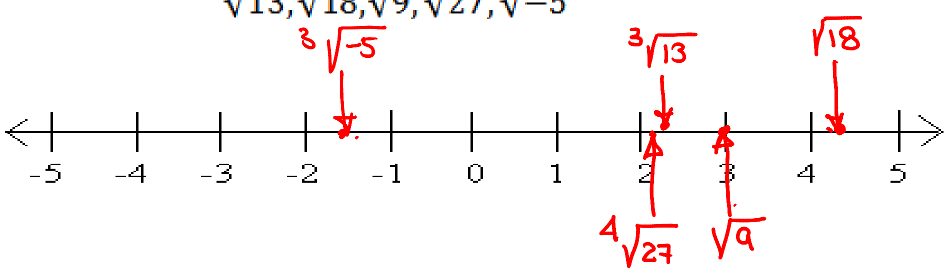
Example 4: *Use a number line to order these numbers from least to greatest.*

$\sqrt[3]{1} = 1$
$\sqrt[3]{8} = 2$
$\sqrt[3]{27} = 3$

$\sqrt[4]{1} = 1$
$\sqrt[4]{16} = 2$
$\sqrt[4]{81} = 3$

$\sqrt[3]{13}, \sqrt{18}, \sqrt{9}, \sqrt[4]{27}, \sqrt[3]{-5}$

N.C. Question



Assignment: p. 211 Q #3–5,7,11,12,14, 15,17–19

Table of Perfect Squares			Table of Perfect Cubes		
$1^2 = 1$	$5^2 = 25$	$9^2 = 81$	$1^3 = 1$	$5^3 = 125$	$9^3 = 729$
$2^2 = 4$	$6^2 = 36$	$10^2 = 100$	$2^3 = 8$	$6^3 = 216$	$10^3 = 1000$
$3^2 = 9$	$7^2 = 49$	$11^2 = 121$	$3^3 = 27$	$7^3 = 343$	$11^3 = 1331$
$4^2 = 16$	$8^2 = 64$	$12^2 = 144$	$4^3 = 64$	$8^3 = 512$	$12^3 = 1728$